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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Statistics

Core Course – V

ST 1541 : LIMIT THEOREMS AND SAMPLING DISTRIBUTIONS

(2022 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions, each question carries **1** mark.

1. What is meant by the limit of a sequence of events?

2. How is a probability measure defined in probability theory?

3. Define convergence in distribution of sequence of random variables.

4. What is the meaning of convergence in probability, and how is it formally defined?

5. Define a random sample in the context of statistics.

6. How would you define a statistic?

7. In which scenario would a t-test be preferred over a z-test for hypothesis testing?

P.T.O.

8. Write the mean and variance of the F-distribution.

9. How is the empirical distribution function defined?

10. Write the probability density function (pdf) of the r^{th} order statistic.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions, each question carries **2** marks.

11. State and explain the monotone property of probability measures.

12. Describe the concepts of limit supremum (limsup) and limit infimum (liminf) of a sequence of events.

13. State the Central Limit Theorem.

14. State Chebychev's weak law of large numbers.

15. Distinguish between standard error and standard deviation.

16. Let $X_n \to X$, in probability. Then for any a > 0, show that $aX_n \to aX$ in probability.

17. Define the Student's t-distribution.

18. How is the noncentrality parameter in the noncentral t-distribution defined?

19. Define the non-central χ^2 distribution.

20. What is an order statistic? Provide an example for a sample of size 5.

21. What are the applications of order statistics?

22. Discuss the continuity property of probability measures.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions, each carries **4** marks.

23. Scores on a test have an average of 60 and a standard deviation of 12. Let S be a score picked at random. Find the best lower and upper bounds you can on $P(S \geq 90)$.

24. State and prove Bernoulli Law of large numbers.

25. Write any four properties of convergence in distribution.

26. State and prove Lindberg-Levy Central Limit theorem.

27. Derive the sampling distribution of the mean for a sample drawn from a normal distribution.

28. State and prove additive property of χ^2.

29. Derive the mean and variance of the χ^2 distribution with n degrees of freedom.

30. Describe the inter-relationships between the F-statistic, t statistic and the chi-square statistic.

31. Obtain the probability density function of the n^{th} order statistic.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions, each carries **15** marks.

32. (a) State and prove Borel-Cantelli lemma.

    (b) State and prove Chebyshev's inequality.

33. (a) Derive the moment generating function (mgf) of the χ^2 distribution.

    (b) Derive sampling distribution of the variance of a sample arising from a normal distribution.

34. Derive the probability distribution and moments of 1^{th} and n^{th} order statistics from $U(0, \theta)$.

35. (a) Show that the r^{th} order statistic in an exponential distribution has a Beta distribution in the form of Beta $(r, n - r + 1)$.

(b) Also deduce pdf for the n^{th} (Maximum) order statistic.

(2 × 15 = 30 Marks)

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Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1542 : ESTIMATION

(2022 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions, Each question carries **1** mark.

1. Define an estimator and an estimate.
2. Define consistency in an estimator.
3. State Cramer–Rao inequality.
4. Define Minimum Variance Bound Estimator.
5. What is meant by confidence coefficient?
6. Write the 95% confidence interval for variance of normal population.
7. What is the method of moments in estimation?
8. Describe the method of least squares.

P.T.O.

9. What are the key assumptions of the Gauss–Markov model in the context of linear estimation?

10. Define the concept of a linear estimator.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions, each question carries **2** marks

11. Explain the difference between a parameter and a statistic.

12. Given a random sample $X_1, X_2, \ldots X_n$ from a *Binomial*(1, *p*) distribution, find an unbiased estimator for p.

13. Define relative efficiency.

14. State the Factorization Theorem and explain how it is used to determine sufficiency of a statistic.

15. Explain the difference between a point estimate and an interval estimate.

16. A sample of 25 observations from a normal population gives a sample mean of 35 and a sample standard deviation of 6. Construct a 90% confidence interval for the population mean.

17. A sample of 30 observations from a normal population is used to estimate the population variance, which is found to be 36. Construct a 95% confidence interval for the population variance.

18. Based on a sample of size n, obtain the method of moments estimator for the parameter p of a Bernoulli distribution.

19. What does it mean for a parametric function to be estimable in the context of a linear model?

20. What is the BLUE (Best Linear Unbiased Estimator)?

21. Write the necessary and sufficient condition for a parametric function is said to be estimable.

22. Explain the unbiasedness property of estimator with example.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions, each carries **4** marks

23. Given $X_1, X_2, \ldots, X_n$ are i.i.d. from a $N(\mu, \sigma^2)$ distribution with unknown σ^2, show that $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is an unbiased estimator of σ^2.

24. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. from an exponential distribution with rate parameter λ. Show that $\hat{\lambda} = \frac{1}{\overline{X}}$ is a consistent estimator of λ.

25. Based a random sample of size n from a Poisson distribution with mean λ, find the MVUE for λ.

26. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. Show that the sample mean is a sufficient statistic for μ.

27. Construct a confidence interval for the population proportion-based on binomial distribution.

28. Based on a sample of size n, estimate the parameters of a uniform distribution on the interval $[a, b]$ using the method of moments.

29. Use the method of maximum likelihood to estimate the rate parameter λ of an exponential distribution from a sample of size n.

30. State Gauss–Markov theorem. Write its applications.

31. Consider the following three linear parametric equations with $E(\epsilon_i) = 0$ for all $i = 1,2,3$

and $Cov(\epsilon_i, \epsilon_j) = \begin{cases} \sigma^2; \text{ if } i = j; \\ 0 \text{ if its } \beta_1 + 2\beta_2 + \epsilon_1 = y_1; \ \beta_2 - 3\beta_3 + \epsilon_2 = y_2; \ 2\beta_1 + \beta_3 + \epsilon_3 = y_3 \end{cases}$

Is the linear function $\beta_1 + \beta_2 + \beta_3$ estimable from the above equations? Justify your answer.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions, each carries **15** marks

32. (a) Given a random sample $X_1, X_2, \ldots, X_n$, from $Binomial(m, p)$ distribution, find an unbiased estimator for p and which is consistent.

(b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. from $Uniform\ (0, \theta)$. Show that the sample mean $\overline{X}$ is unbiased and consistent estimator for $\frac{\theta}{2}$.

33. For a random sample from an exponential distribution Exp(λ), find the maximum likelihood estimator for λ and check whether it is consistent or not.

34. Construct a confidence interval for the difference between the means of two independent normal populations.

35. Derive the maximum likelihood estimator for the mean and variance of a normal distribution based on a sample of size n. Show that these estimators are consistent.

(2 × 15 = 30 Marks)

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Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 : TESTING OF HYPOTHESIS

(2022 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions, each question carries **1** mark

1. What is a statistical hypothesis?
2. How is the power of a test defined?
3. What is a UMP test?
4. Write any two procedures to derive a test for a statistical hypothesis.
5. Which distribution is used for testing the standard deviation of a normal population?
6. What is the degrees of freedom for the chi-square test of independence when there are m and n levels for each variable?
7. What test is used to compare the means of two normal population in the case of small samples.
8. Write the test statistics used to test for the equality of variances of two normal populations.
9. Define the empirical distribution function.
10. Provide an example of a U-statistic.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions, each question carries **2** marks

11. How does a simple hypothesis differ from a composite hypothesis?
12. Distinguish between Type-I error and Type-II error.
13. What is the difference between power curve and power function?
14. Define critical region.
15. State Neyman-Pearson lemma.
16. Write the test statistic for testing the mean of a binomial distribution using Neyman-Pearson's lemma.
17. Write the test statistic for testing the equality of two proportions.
18. Write the test statistic for testing significance of difference between two correlation coefficients.
19. Explain the concept of non-parametric estimation. How does it differ from parametric estimation, and in what situations is it preferred?
20. What is a U-statistic, and why is it important in non–parametric estimation?
21. Define the "degree of an estimable parameter" and explain its significance in non–parametric statistics.
22. Distinguish between level of significance and size of a test.

($8 \times 2 = 16$ Marks)

SECTION – C

Answer any **six** question, each carries **4** marks

23. Distinguish between most powerful test, uniformly most powerful test.
24. Derive the test for the mean of a population following a Poisson distribution.
25. State the likelihood ratio test and list its properties.
26. Explain the method for testing the equality of means of two populations means in the case of large samples.
27. A study investigates the relationship between hours studied and exam scores among 30 students. The sample correlation coefficient between hours studied and exam scores is $r = 0.45$. Test the significance of this correlation coefficient at a 5% significance Level.

28. A researcher records the number of accidents occurring at a busy intersection over 100 days and observes the following frequency distribution of accidents per day:

Number of Accidents (X)	Frequency (Observed)
0	40
1	30
2	20
3	8
4 or more	2

Assuming that the number of accidents follows a Poisson distribution, use the chi-square goodness-of-fit test to determine if the observed data fits a Poisson distribution at a 5% significance level. Use the mean number of accidents as the parameter for the Poisson distribution.

29. A sample of 10 observations is drawn from a normal population with an unknown mean. The sample mean is 25, and the sample standard deviation is 4. Test whether the population mean is significantly different from 20 at the 5% significance level.

30. Two samples are drawn from two normal populations. Sample A has a mean of 18 with a standard deviation of 3 (n = 15), and Sample B has a mean of 22 with a standard deviation of 4 (n=15). Test if the means of the two populations are significantly different at a 1% significance level.

31. Two machines produce parts with sample standard deviations of 5 and 8, respectively. The sample sizes of samples from Machine A and Machine B are 10 and 12. Test if the variances of the two populations are significantly different at a 5% significance level.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions, each carries **15** marks 32-35

32. Derive most powerful test for the mean of a normal population when the variance is unknown.

33. A company wants to investigate whether there is significant difference in the average productivity scores of employees working from home and those working in the office. The productivity scores for a sample of employees in each group are as follows:

Work-from-home Productivity Scores (25 observations):

8,7,9,6,8,7,10, 6,7,9,8,7,9,6,8,7,10,8,9,7.6,8,9,7, 10

In-office Productivity Scores (32 observations):

6, 7, 5, 8, 6. 7, 9, 8, 5, 7, 6, 7, 8, 6, 7, 8, 5, 6, 8, 7, 6, 5, 8, 6, 7, 8. 5, 7, 6, 7, 6, 8.

Using a significance level of 5%, test if there is a significant difference in the mean productivity scores of these two groups. Assume the population variances are unknown but similar.

34. A study examines the relationship between exercise duration (in hours per week) and blood pressure reduction (in mmHg) for 6 participants. Given the following data, test if the correlation between exercise duration and blood pressure reduction is significantly different from zero at the 10% significance level.

Participant	Exercise Duration (hours)	Blood Pressure Reduction (mmHg)
1	5	10
2	8	15
3	3	5
4	10	20
5	2	3
6	6	12

Given correlation coefficient $r = -0.6$

35. Write note on the following:

(a) Mann-Whitney-Wilcoxon test

(b) Kolmogorov-Smirnov one-sample test

(2 × 15 = 30 Marks)

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Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 : SAMPLE SURVEY METHODS

(2022 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. What is a sampling unit?

2. Define sampling frame.

3. What you mean by finite population corrector?

4. Define linear regression estimator of population mean.

5. Which sampling method will be adopted to select a cricket team of a country for the world cup?

6. What are sampling interval and random start in systematic sampling?

7. How is the sampling error reduced in a survey?

P.T.O.

8. What is the number of possible samples of size 2 out of population size 5 in SRSWR?

9. What is allocation in sampling theory?

10. Define ratio estimator of population mean under simple random sampling.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. What do you understand by population in statistical sense? Give an example.

12. What is the difference between SRSWOR and SRSWR?

13. Define proportional allocation in stratification.

14. What is circular systematic sampling?

15. Find an unbiased estimator of population mean in stratified random sampling.

16. What are the circumstances under which census surveys are preferred over sample surveys?

17. With usual notations, prove that $B(\hat{R}) = \frac{-Cov(\hat{R}, \bar{x})}{\bar{X}}$.

18. Point out the merits of random sampling.

19. Show that sample proportion is an unbiased estimator of population proportion in SRSWOR of attributes.

20. Suppose a population consists of 5 units : 2, 3, 6, 8, 11. Find the possible samples of size 2 from the population following SRSWOR.

21. Under what situations would you use systematic sampling?

22. Give any two advantages of stratification.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Explain briefly the advantages of sampling over census.

24. In stratified random sampling, find the sample size for each stratum under optimum allocation.

25. Obtain confidence interval of population mean in a simple random sampling.

26. In simple random sampling without replacement, for large sample size, find an approximation to variance of ratio estimator.

27. Derive the formula for sample size in random sampling for proportions.

28. What is judgment sampling? What are its merits and limitations?

29. Show that for a simple random sample without replacement, with usual notations, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ is an unbiased estimator of $S^2 = \frac{1}{N-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$.

30. Explain systematic random sampling. Point out the merits of systematic sampling.

31. Explain the comparison of ratio estimator of population total with the estimator based on mean per unit in simple random sampling.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Derive the expression for the variance of estimator of population mean in SRSWOR. Compute the efficiency of the estimator under SRSWOR over SRSWR.

    (b) What are sampling and non sampling errors? Give the main sources of bias in a sample survey. What are the sources of non sampling errors?

33. Describe the basic steps in planning and execution of a sample survey.

34. Explain stratified random sampling. Compare the precision of stratified sampling under proportional and optimum allocations with simple random sampling.

35. (a) Under linear systematic sampling$_1$ find an unbiased estimator of population mean and derive the variance of the estimator of population mean. Show that systematic sampling is more precise than SRS if the variance within systematic sampling is larger than the population variance as a whole.

(b) Show that in a population with linear trend, systematic sampling is less efficient than stratified random sampling for the estimation of population mean.

(2 × 15 = 30 Marks)
